THE $SL(2)$-TYPE AND BASE CHANGE

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Abstract. The $SL(2)$-type of any smooth, irreducible and unitarizable representation of $GL_n$ over a $p$-adic field was defined by Venkatesh. We provide a natural way to extend the definition to all smooth and irreducible representations. For unitarizable representations we show that the $SL(2)$-type of a representation is preserved under base change with respect to any finite extension. The Klyachko model of a smooth, irreducible and unitarizable representation $\pi$ of $GL_n$ depends only on the $SL(2)$-type of $\pi$. As a consequence we observe that the Klyachko model of $\pi$ and of its base-change are of the same type.

1. INTRODUCTION

Let $F$ be a finite extension of $\mathbb{Q}_p$. In [Ven05], Venkatesh assigned a partition of $n$, the $SL(2)$-type of $\pi$, to any smooth, irreducible and unitarizable representation $\pi$ of $GL_n(F)$. For a representation of Arthur type the $SL(2)$-type encodes the combinatorial data in the Arthur parameter. In general, the $SL(2)$-type is defined in terms of Tadic's classification of the unitary dual.

The reciprocity map for $GL_n(F)$ is a bijection from the set of isomorphism classes of smooth irreducible representations of $GL_n(F)$ to the set of isomorphism classes of $n$-dimensional Weil-Deligne representations (cf. [HT01] and [Hen00]). Applying the reciprocity map we observe that there is a natural way to extend the definition of the $SL(2)$-type to all smooth and irreducible representations of $GL_n(F)$ (see Theorem 4.1 and Remark 1). The reciprocity map also allows the definition of base change with respect to any finite extension $E$ of $F$. It is a map $bc_{E/F}$ from isomorphism classes of smooth irreducible representation of $GL_n(F)$ to isomorphism classes of smooth irreducible representation of $GL_n(E)$ that is the ‘mirror image’ of restriction with

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respect to $E/F$ of Weil-Deligne representations. The content of Theorem 6.1, our main result, is that for any smooth, irreducible and unitarizable representation $\pi$ of $GL_n(F)$ the representations $\pi$ and $bc(\pi)$ have the same $SL(2)$-type.

In [OS07b], [OS07a], [OS08] we studied the Klyachko models of smooth irreducible representations of $GL_n(F)$, that is, distinction of a representation with respect to certain subgroups that are a semi direct product of a unipotent and a symplectic group. Our results are also described in terms of Tadic’s classification and depend, in fact, only on the $SL(2)$-type of a representation. For example, a smooth, irreducible and unitarizable representation $\pi$ of $GL_2(F)$ is $Sp_2(F)$-distinguished, i.e. it satisfies $\text{Hom}_{Sp_2(F)}(\pi, \mathbb{C}) \neq 0$, if and only if the $SL(2)$-type of $\pi$ consists entirely of even parts (and in this case $\text{Hom}_{Sp_2(F)}(\pi, \mathbb{C})$ is one dimensional [HR90, Theorem 2.4.2]). For unitarizable representations, our results on Klyatchko models are reinterpreted here in terms of the $SL(2)$-type. As a consequence we show that Klyachko models are preserved under base-change with respect to any finite extension.

In particular, we have

**Theorem 1.1.** Let $E/F$ be a finite extension of $p$-adic fields. A smooth, irreducible and unitarizable representation $\pi$ of $GL_{2n}(F)$ is $Sp_{2n}(F)$-distinguished if and only if $bc_{E/F}(\pi)$ is $Sp_{2n}(E)$-distinguished.

The rest of this note is organized as follows. After setting some general notation in Section 2, in Section 3 we recall the definition of the reciprocity map. In Section 4 we recall the definition of Venkatesh for the $SL(2)$-type of a unitarizable representation and extend it to all smooth irreducible representations. We recall (and reformulate in terms of the $SL(2)$-type) our results on symplectic (and more generally on Klyachko) models in Section 5. Our main observation Theorem 6.1 and its application to Klyachko models Corollary 6.1 are stated in Section 6 and proved in Section 7. The main theorem says that base change respects $SL(2)$-types and its corollary says that base change respects Klyachko types. Theorem 1.1 is a special case where the Klyachko type is purely symplectic.

**2. Notation**

Let $F$ be a finite extension of $Q_p$ for some prime number $p$ and let $| \cdot_F : F^\times \to \mathbb{C}^\times$ denote the standard absolute value normalized so that the inverse of uniformizers are mapped to the size of the residual field. Denote by $W_F$ the Weil group of $F$ and by $I_F$ the inertia subgroup of $W_F$. We normalize the reciprocity map $T_F : W_F \to F^\times$, given by local class field theory, so that geometric Frobenius elements are mapped to
uniformizers. The map $T_F$ defines an isomorphism from the abelianization $W^\text{ab}_F$ of $W_F$ to $F^\times$ (this is the inverse of the Artin map). Let $|\cdot|_{W_F} = |\cdot|_F \circ T_F$ denote the associated absolute value on $W_F$.

Denote by $1_\Omega$ the characteristic function of a set $\Omega$. Let $\text{MS}_\text{fin}(\Omega)$ be the set of finite multisets of elements in $\Omega$, that is, the set of functions $f : \Omega \to \mathbb{Z}_{\geq 0}$ of finite support. When convenient we will also denote $f$ by $\{\omega_1, \ldots, \omega_1, \omega_2, \ldots, \omega_2, \ldots\}$ where $\omega \in \Omega$ is repeated $f(\omega)$ times.

Let $P = \text{MS}_\text{fin}(\mathbb{Z}_{>0})$ be the set of partitions of positive integers and let $P(n) = \{ f \in P : \sum_{k=1}^{\infty} k f(k) = n \}$ denote the subset of partitions of $n$. For $n, m \in \mathbb{Z}_{>0}$ let $(n)_m = m \, 1_n = \{n, \ldots, n\}$ be the partition of $nm$ with $m$ parts of size $n$. Let $\text{odd} : P \to \mathbb{Z}_{\geq 0}$ be defined by

$$\text{odd}(f) = \sum_{k=0}^{\infty} f(2k + 1),$$

i.e. $\text{odd}(f)$ is the number of odd parts of the partition $f$.

3. Reciprocity and base-change for $GL_n(F)$

3.1. Weil-Deligne representations. An $n$-dimensional Weil-Deligne representation is a pair $((\rho, V), N)$ where $(\rho, V)$ is an $n$-dimensional representation of $W_F$ that decomposes as a direct sum of irreducible representations and $N : V \to V$ is a linear operator such that

$$|w|_{W_F} \, N \circ \rho(w) = \rho(w) \circ N, \quad w \in W_F.$$

The map $((\rho, V), N) \mapsto ([\rho], f)$, where $[\rho]$ denotes the isomorphism class of the $n$-dimensional representation $(\rho, V)$ of $W_F$ and $f \in P(n)$ is the partition of $n$ associated to the Jordan decomposition of $N$, defines an injective map on isomorphism classes of Weil-Deligne representations. Denote its image by $G_F(n)$. In this way we identify the set $G_F(n)$ with the set of isomorphism classes of $n$-dimensional Weil-Deligne representations. Let $P_{F,n} : G_F(n) \to P(n)$ be the projection to the second coordinate. Let $G_F = \bigcup_{n=1}^{\infty} G_F(n)$ be the set of isomorphism classes of all finite dimensional Weil-Deligne representations and let $P_F : G_F \to P$ be the map such that $P_F|_{G_F(n)} = P_{F,n}$.

3.2. The local Langlands correspondence. Let $A_F(n)$ be the set of isomorphism classes of smooth and irreducible representations of $GL_n(F)$ and set $A_F = \bigcup_{n=1}^{\infty} A_F(n)$. For every $\pi \in A_F$ we denote by $\omega_\pi$ the central character of (any representation in the isomorphism class of) $\pi$. Fix a non trivial additive character $\psi$ of $F$. Due to Harris-Taylor
[HT01] and independently to Henniart [Hen00] there exists a unique sequence of bijections
\[ \text{rec}_{F,n} : A_F(n) \to G_F(n) \]
for all \( n \geq 1 \) satisfying the following properties:

\begin{align*}
(3.1) \quad & \text{rec}_F(\chi) = \chi \circ T_F; \\
(3.2) \quad & L(\pi_1 \times \pi_2, s) = L(\text{rec}_F(\pi_1) \otimes \text{rec}_F(\pi_2), s); \\
(3.3) \quad & \epsilon(\pi_1 \times \pi_2, s, \psi) = \epsilon(\text{rec}_F(\pi_1) \otimes \text{rec}_F(\pi_2), s, \psi); \\
(3.4) \quad & \det \circ \text{rec}_F(\pi) = \text{rec}_F(\omega_\pi); \\
(3.5) \quad & \text{rec}_F(\pi^\vee) = \text{rec}_F(\pi)^\vee.
\end{align*}

Here \( \chi \in A_F(1), \pi, \pi_1, \pi_2 \in A_F, \pi^\vee \) is the contragredient of \( \pi, \text{rec}_F(\pi)^\vee \) is the dual of \( \text{rec}_F(\pi) \) and \( \text{rec}_F : A_F \to G_F \) is such that \( \text{rec}_{F|A_F(n)} = \text{rec}_{F,n} \).

3.3. **Expressing \( \text{rec}_F \) in terms of \( \text{rec}_F^o \).** Let \( A_F^o(n) \subseteq A_F(n) \) be the subset of isomorphism classes of supercuspidal representations and let \( G_F^o(n) \subseteq G_F(n) \) be the subset of isomorphism classes \(([\rho], f)\) such that \( \rho \) is irreducible and \( f = 1_n = \{n\} \). The set \( G_F^o(n) \) is identified with the set of isomorphism classes of irreducible and \( n \)-dimensional representations of \( W_F \). It follows from the work of Harris-Taylor and independently of Henniart that there exists a unique sequence of bijections
\[ \text{rec}_{F,n|A_F^o(n)} = \text{rec}_{F,n}^o : A_F(n) \to G_F^o(n) \]
satisfying (3.1), (3.2), (3.3), (3.4) and (3.5). The work of Zelevinsky [Zel80] allows the extension of \( \text{rec}_F^o \) to the map \( \text{rec}_F \) on \( A_F \). This is also explained in [Hen85] and we now recall the construction of \( \text{rec}_F \) in terms of \( \text{rec}_F^o \).

For \( s \in \mathbb{C} \) and every isomorphism class \( \varpi = [\pi] \in A_F \) (resp. \( \varrho = ([\rho], f) \in G_F \) ) let \( \varpi[s] = [\pi \otimes |\det|_F^o] \) (resp. \( \varrho[s] = ([\rho \otimes |\det|_{W_F}^o], f) \) ). A segment in \( A_F^o \) (resp. \( G_F^o \) ) is a set of the form
\[ \Delta[\sigma, r] = \{\sigma[\frac{1-r}{2}], \sigma[\frac{3-r}{2}], \ldots, \sigma[\frac{r-1}{2}] \} \]
isomorphism class \( \varpi = [\pi] \in A_F \) (resp. \( \varrho = ([\rho], f) \in G_F \) ) and \( r \in \mathbb{Z}_{>0} \). Let \( S \) (resp. \( S' \) ) denote the set of all segments in \( A_F^o \) (resp. \( G_F^o \) ) and let \( \mathcal{O} = \text{MS}_{\text{fin}}(S) \) (resp. \( \mathcal{O}' = \text{MS}_{\text{fin}}(S') \) ). The bijection \( \text{rec}_F^o : A_F^o \to G_F^o \) defines a bijection \( \text{rec}_F^o : \mathcal{S} \to S' \) given by \( \text{rec}_F^o(\Delta[\sigma, r]) = \Delta[\text{rec}_F^o(\sigma), r] \) and a bijection \( \text{rec}_F^o : \mathcal{O} \to \mathcal{O}' \) given by \( \text{rec}_F^o(a)(\text{rec}_F^o(\Delta)) = a(\Delta), \Delta \in S \).
In [Zel80, Section 6.5] Zelevinsky defines a bijection \( a \mapsto \langle a \rangle \) from \( \mathcal{O} \) to \( \mathcal{A}_F \). The Zelevinsky involution is defined in [Zel80, Section 9.12] as an involution on the Grothendieck group associated with \( \mathcal{A}_F \). It is proved by Aubert [Aub95], [Aub96] and independently by Procter [Pro98] that the Zelevinsky involution restricts to a bijection from \( \mathcal{A}_F \) to itself that we denote by \( \pi \mapsto \pi^t \). In [Zel80, Section 10.2] Zelevinsky defines a bijection \( \tau : \mathcal{O}' \to \mathcal{G}_F \) as follows. For a segment \( \Delta[\rho, r] \in \mathcal{S}' \) where \( \rho \in \mathcal{G}^\circ \mathcal{F}(t) \) let

\[
\tau(\Delta[\rho, r]) = (\oplus_{i=1}^r \rho_i(r)_t)
\]

and for \( a' \in \mathcal{O}' \) set

\[
\tau(a') = \oplus_{\Delta' \in \mathcal{O}} \tau(\Delta')
\]

where for \((\rho_1, f_1), \ldots, (\rho_m, f_m) \in \mathcal{G}_F \) the direct sum is given by

\[
(\rho_1 \oplus \cdots \oplus \rho_m, f_1 + \cdots + f_m)
\]

The reciprocity map \( \text{rec}_F \) is given by

\[
\text{rec}_F(\langle a \rangle^t) = \tau(\text{rec}_F(a)), \ a \in \mathcal{O}.
\]

4. The \( SL(2) \)-type of a representation

Denote by \( \mathcal{A}_F^\circ(n) \) the subset of \( \mathcal{A}_F(n) \) consisting of all isomorphism classes of unitarizable representations and let \( \mathcal{A}_F^u = \bigcup_{n=1}^\infty \mathcal{A}_F(n) \). For \([\pi_1], \ldots, [\pi_m] \in \mathcal{A}_F \) we denote by \( \pi_1 \times \cdots \times \pi_m \) the representation parabolically induced from \( \pi_1 \otimes \cdots \otimes \pi_m \) and by \([\pi_1] \times \cdots \times [\pi_m] \) its isomorphism class.

For \( \sigma \in \mathcal{A}_F^\circ \) and integers \( n, r > 0 \) let

\[
\delta[\sigma, n] = \langle \Delta[\sigma, n] \rangle^t,
\]

\[
a(\sigma, n, r) = \{ \Delta[\sigma[\frac{1-r}{2}], n], \Delta[\sigma[\frac{3-r}{2}], n], \cdots, \Delta[\sigma, (\frac{r-1}{2})] \} \in \mathcal{O}
\]

and

\[
U(\delta[\sigma, n], r) = \langle a(\sigma, n, r) \rangle
\]

Tadic’s classification of the unitary dual of \( GL_n(F) \) [Tad86] implies that if \( \sigma \in \mathcal{A}_F^\circ \cap \mathcal{A}_F^u \) then \( U(\delta[\sigma, n], r) \in \mathcal{A}_F^u \) and that for any \( \pi \in \mathcal{A}_F^u \) there exist \( \sigma_1, \ldots, \sigma_m \in \mathcal{A}_F^\circ \) and integers \( n_1, \ldots, n_m, r_1, \ldots, r_m > 0 \) such that

\[
(4.1) \quad \pi = U(\delta[\sigma_1, n_1], r_1) \times \cdots \times U(\delta[\sigma_m, n_m], r_m).
\]

It further follows from [Tad95, Lemma 3.3] that

\[
(4.2) \quad U(\delta[\sigma, n], r)^t = U(\delta[\sigma, r], n).
\]
The $SL(2)$ of a representation $\pi \in A_F$ of the form (4.1) is defined in [Ven05, Definition 1] to be the partition

\[ \{(r_1)_{n_1}, \ldots, (r_m)_{n_m}\}. \]

**Theorem 4.1.** The $SL(2)$-type of a representation $\pi \in A_F$ equals $P_F(\text{rec}_F(\pi^t))$.

**Remark 1.** Theorem 4.1 allows us to define the $SL(2)$-type of any $\pi \in A_F$ by the formula $P_F(\text{rec}_F(\pi^t))$. Note further that given a reciprocity map (local Langlands conjecture), this provides a recipe to define the $SL(2)$-type of an irreducible representation for any reductive group!

**Proof.** Based on Tadic’s classification of the unitary dual of $GL_n(F)$, the proof of Theorem 4.1 is merely a matter of following the definitions. For convenience, we provide the proof. The key is in the following simple observations.

**Lemma 4.1.** Let $\pi \in A_F$ be of the form (4.1). Then

\[ \text{rec}_F(\pi) = \bigoplus_{i=1}^m \bigoplus_{j=1}^{r_i} \tau(\Delta[\sigma_i]^{r_i+1}/2 - j, n_i) \]

and

\[ \pi^t = U(\delta[\sigma_1, r_1], n_1) \times \cdots \times U(\delta[\sigma_m, r_m], n_m) \in A_F. \]

**Proof.** Let $a_i = a(\sigma_i, r_i, n_i)$. It follows from (4.2) that

\[ \pi = \langle a_1 \rangle^t \times \cdots \times \langle a_m \rangle^t = (\langle a_1 \rangle \times \cdots \times \langle a_m \rangle)^t \]

and since $t$ is an involution on $A_F$ that $\langle a_1 \rangle \times \cdots \times \langle a_m \rangle \in A_F$. Thus, it follows from [Zel80, Proposition 8.4] that $\langle a_1 \rangle \times \cdots \times \langle a_m \rangle = \langle a_1 + \cdots + a_m \rangle$. In other words $\pi = \langle a_1 + \cdots + a_m \rangle^t$ and therefore by definition

\[ \text{rec}_F(\pi) = \tau(\text{rec}_F^\circ(a_1 + \cdots + a_m)) = \bigoplus_{i=1}^m \tau(\text{rec}_F^\circ(a_i)). \]

The identity (4.4) now follows from the definition of $\tau(\text{rec}_F^\circ(a_i))$. Note that (4.2) implies that

\[ \pi^t = U(\delta[\sigma_1, r_1], n_1) \times \cdots \times U(\delta[\sigma_m, r_m], n_m) \]

and the classification of Tadic therefore implies that $\pi^t \in A_F$. Thus we get (4.5).

Applying (4.4) to $\pi^t$ and comparing with (4.3) Theorem 4.1 follows from the definitions. □

From now on for every $\pi \in A_F$ we denote by

\[ V(\pi) = P_F(\text{rec}_F(\pi^t)) \]

the $SL(2)$-type of $\pi$. 

5. Klyachko models

For positive integers \( r \) and \( k \) denote by \( U_r \) the subgroup of upper triangular unipotent matrices in \( GL_r(F) \) and by \( Sp_{2k}(F) \) the symplectic group in \( GL_{2k}(F) \). Fix a decomposition \( n = r + 2k \). Let

\[
H_{r,2k} = \{ \begin{pmatrix} u & X \\ 0 & h \end{pmatrix} : u \in U_r, X \in M_{r \times 2k}(F), h \in Sp_{2k}(F) \}.
\]

Let \( \psi \) be a non trivial character of \( F \). For \( u = (u_{i,j}) \in U_r \) let

\[
\psi_r(u) = \psi(u_{1,2} + \cdots + u_{r-1,r})
\]

and let \( \psi_{r,2k} \) be the character of \( H_{r,2k} \) defined by

\[
\psi_{r,2k} \begin{pmatrix} u & X \\ 0 & h \end{pmatrix} = \psi_r(u).
\]

We refer to the space

\[
M_{r,2k} = \text{Ind}_{H_{r,2k}}^{GL_n(F)}(\psi_{r,2k})
\]

as a Klyachko model for \( GL_n(F) \). Here \( \text{Ind} \) denotes the functor of non-compact smooth induction.

In [OS08, Corollary 1] we showed that for any \( \pi \in A_u^u(F) \) there exists a unique decomposition

\[
n = r(\pi) + 2k(\pi)
\]

such that

\[
\text{Hom}_{GL_n(F)}(\pi, M_{r(\pi),2k(\pi)}) \neq 0
\]

and that in fact \( \dim_{\mathbb{C}}(\text{Hom}_{GL_n(F)}(\pi, M_{r(\pi),2k(\pi)})) = 1 \).

Definition 1. For \( \pi \in A_u^u(F) \), the Klyachko type of \( \pi \) is the ordered pair \((r(\pi),2k(\pi))\).

In fact, for \( A_u^u(F) \) [OS07a, Theorem 8] provides a receipt in order to read the Klyachko type off Tadic’s classification. Based on (4.3), our results can be reinterpreted by the formula

\[
r(\pi) = \text{odd}(\mathcal{V}(\pi)), \quad \pi \in A_u^u(F).
\]

6. Base change-The main results

Let \( E \) be a finite extension of \( F \). Denote by \( \text{res}_{E/F,n} : G_F(n) \rightarrow G_E(n) \) the map defined by \( \text{res}_{E/F,n}([\rho],f) = ([\rho|_{W_E}], f) \). For \( n \geq 1 \) the base change \( \text{bc}_{E/F}(\pi) \in A_E(n) \) of \( \pi \in A_F(n) \) is defined by

\[
\text{rec}_E(\text{bc}_{E/F}(\pi)) = \text{res}_{E/F}(\text{rec}_F(\pi)).
\]
Theorem 6.1. Let $E/F$ be a finite extension of $p$-adic fields and let $\pi$ be a smooth, irreducible and unitarizable representation of $GL_n(F)$. Then $bc_{E/F}(\pi)$ is a smooth, irreducible and unitarizable representation of $GL_n(E)$ and

$$V(\pi) = V(bc_{E/F}(\pi)),$$

i.e. $\pi$ and $bc_{E/F}(\pi)$ have the same $SL(2)$-type.

As a consequence we have the following.

Corollary 6.1. Under the assumptions of Theorem 6.1 we have

$$r(\pi) = r(bc_{E/F}(\pi)),$$

i.e. $\pi$ and $bc_{E/F}(\pi)$ have the same Klyachko type.

Corollary 6.1 is straightforward from Theorem 6.1 and (5.1).

7. Proof of the main result

Lemma 7.1. Let $E/F$ be a finite extension. For $\sigma \in A_F^0 \cap A_F^u$ there exist $\sigma_1, \ldots, \sigma_m \in A_E^0 \cap A_E^u$ such that

$$bc_{E/F}(\sigma) = \sigma_1 \times \cdots \times \sigma_m.$$

Proof. Recall that a representation in $A_F^0$ is unitarizable if and only if its central character is unitary. Let $\rho$ be the irreducible representation of $W_F$ such that $\text{rec}_F(\sigma) = ([\rho], 1_n)$. It follows from (3.4) that $\rho$ has a unitary central character and therefore it has a unitary structure. Thus, the restriction $\rho_{|W_E}$ to $W_E$ also has a unitary structure and therefore each of its irreducible components has a unitary central character. The lemma follows by applying (4.4) to $\text{res}_{E/F}(\text{rec}_F(\sigma))$. \hfill \Box

Proposition 7.1. Let $E/F$ be a finite extension and let $\pi \in A_F^u$ then $bc(\pi) \in A_E^u$ and

$$bc_{E/F}(\pi^t) = bc_{E/F}(\pi)^t.$$

Proof. Let $\pi \in A_F^u$ be of the form (4.1). By Lemma 7.1 there exist $\sigma_{i,k} \in A_E^0, i = 1, \ldots, m$, $k = 1, \ldots, t_i$ such that

$$bc_{E/F}(\sigma_i) = \sigma_{i,1} \times \cdots \times \sigma_{i,t_i}.$$

Let $\rho_i = \text{rec}_F(\sigma_i)$ and $\rho_{i,k} = \text{rec}_E(\sigma_{i,k})$. Thus,

$$\text{res}_{E/F}(\rho_i) = \oplus_{k=1}^{t_i} \rho_{i,k}.$$

It follows from (4.4) that

$$\text{res}_{E/F}(\text{rec}_F(\pi)) = \oplus_{i=1}^m \oplus_{j=1}^{r_i} \oplus_{k=1}^{t_i} \tau(\Delta[\sigma_{i,k}, \frac{r_i+1}{2} - j], n_i)).$$
On the other hand, let
\[ \Pi = \times_{i=1}^{m} \times_{k=1}^{t_i} U(\delta[\sigma_{i,k}, n_i], r_i) \]
Since \( \pi \in \mathcal{A}_F^e \), the classification of Tadic implies that \( \Pi \in \mathcal{A}_E^u \) and by (4.4) applied to \( E \) instead of \( F \) we have
\[ \text{rec}_E(\Pi) = \oplus_{i=1}^{m} \oplus_{j=1}^{r_i} U(\delta[\sigma_{i,k}, r_i], n_i)) \]
Comparing (7.2) with (7.3) we obtain that \( \Pi = bc_{E/F}(\pi) \) and in particular that \( bc_{E/F}(\pi) \in \mathcal{A}_E^u \). Applying this to \( \pi_t \) expressed by (4.5) gives
\[ bc_{E/F}(\pi_t) = \times_{i=1}^{m} \times_{k=1}^{t_i} U(\delta[\sigma_{i,k}, r_i], n_i)) \]
Applying (4.5) now to \( bc_{E/F}(\pi_t) \) we obtain the identity (7.1). \( \square \)

It is straightforward from the definitions that
\[ P_F(\text{rec}_F(\pi)) = P_E(\text{rec}_E(bc_{E/F}(\pi))), \pi \in \mathcal{A}_F \]
For \( \pi \in \mathcal{A}_F^e \), applying (7.4) to \( \pi_t \) and then (7.1) we get that
\[ P_F(\text{rec}_F(\pi_t)) = P_E(\text{rec}_E(bc_{E/F}(\pi_t))) \]
The identity \( \mathcal{V}(\pi) = \mathcal{V}(bc_{E/F}(\pi)) \) is now immediate from (4.6). This completes the proof of Theorem 6.1.

References


